PERGAMON

Mechatronics 13 (2003) 153–174

MECHATRONICS

Towards kinematic classification schemes for planetary surface locomotion systems

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Received 1 July 2000; accepted 2 February 2001

Abstract

As the planetary exploration programme matures, the objectives for robotic missions become more complex, and place greater demands upon the reliability and robustness of the autonomous systems created for these tasks. Consequently, a situation has arisen where the design of mechanical architectures may be insufficiently flexible to take advantage of the latest developments in control techniques. In particular, mechanical systems with sufficient functionality to tackle advanced autonomous operations, frequently exhibit inadequate robustness in the face of inevitable system faults to meet the requisite mission goals. This paper considers ways in which the kinematic structure of planetary exploration vehicles can be partly characterised using Graph Theory techniques. These include interchange graphs, their spanning trees, and their fundamental cycles and cutsets; adjacency matrices; degree sequences, and characteristic polynomials. It goes on to assess how such a characterisation may be used to define those kinematic features which make for successful, robust systems, well fitted to the complex mission goals required. Consideration is given to whether theoretical kinematic classification could provide a worthwhile building block in the development of novel design tools for future autonomous system development. A valid methodology for analysing and representing system characteristics is derived and discussed, and potential for further enhancement of the approach is identified.

1. Introduction

The pressures to establish cost effective and reliable planetary exploration missions are intense at this time. The search for water on the Moon, and the geophysical

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0957-4158/03/$ - see front matter
PII: S0957-4158(01)00038-1

and exobiological characterisation of Mars are key features in the space programmes of Europe, the USA, Russia and Japan. At the same time, the difficulties of running a manned space programme, particularly in terms of cost, the radiation and microparticle environments, and other hazards mean that precursor, robotic missions currently receive considerable attention. However, as the planetary exploration programme matures, the objectives for such robotic missions become more complex, and place greater demands upon the reliability and robustness of the autonomous systems created for these tasks, Hobbs et al. [1]. Typical of currently planned missions where these technologies are paramount is the European Space Agency's BepiColombo mission to Mercury, due for launch in 2009.

Considerable strides have been made in control system design, and in associated software and logic approaches. Algorithmic, state-based, and fuzzy-logic techniques are only some of the main competitors in the search to identify more fit solutions for these problems. Furthermore, traditional analytical, expert system and knowledge-based approaches to design and fault analysis now have rivals in the genetic algorithm/neural network techniques espoused by the evolutionary robotics research community, Harvey et al. [2].

The result is therefore a situation where the design of mechanical architectures is frequently insufficiently flexible to take advantage of the latest developments in control techniques. In particular, mechanical systems with sufficient functionality to tackle advanced autonomous operations, frequently exhibit inadequate robustness in the face of the inevitable system faults to meet the requisite mission goals.

Construction of new design tools is envisaged as a two-pronged development, which, if successful, should provide fresh insights into this problem. On the one hand, the establishment of classification schemes is seen as a step towards the consistent characterisation of practical examples of autonomous systems. On the other hand, it is necessary to identify methods for the synthesis of theoretical systems within a design space evolved by reference to the capabilities of actual systems. This dual approach should provide valuable building blocks for the development of novel design techniques.

This paper considers ways in which the kinematic structure of planetary exploration vehicles can be characterised, and how this characterisation may be used to define those kinematic features which make for successful, robust systems, well fitted to the complex mission goals required. The objective is to establish whether the definition of such a theoretical kinematic classification is possible, and whether it would provide a worthwhile building block in the development of new design tools for future autonomous system development, as described above.

2. Typical systems

For the purposes of this paper, we concentrate purely on the locomotion subsystems of planetary exploration vehicles. Many variants of these exist, although only three have actually seen 'active service' (two on the Moon, one on Mars). A

Table 1
Some representative planetary locomotion systems

Name	Description	Organisation
Lunakhod	Simple chassis, 8 wheel	(USSR)
Apollo LRV	Simple chassis, 4 wheel	NASA/JPL USA
Marsokhod	Advanced chassis, 6 wheel	Lavochkin (USSR)
Sojourner	Rocker bogie chassis, 6 wheel	NASA/JPL USA (Mars Pathfinder)
Go-for	Fork-wheel, 4 wheels	CMU/JPL, USA
MIDD	Deployable simple chassis, 4 wheels	ESA, with DLR, Germany
MOFFIT study	Complex wheeled system	Astrium (UK)
Millennium hero	Biological hexapod (study only)	Astrium (UK)
Attila	Quasi-biological hexapod	MIT, USA
IOAN	Articulated quasi-biological hexapod	Université Libre de Bruxelles
Sussex octopod	Quasi-biological octopod	University of Sussex, UK
Walkie-6	Twin-frame geometrical hexapod	Alenia, University of Turin, ESA
Waroma	Flipper	European space agency
Nanokhod – original	Crawler	ESA & Max-Planck Institut für Chemie
Nanokhod – current	Tracked	ESA & Max-Planck Institut für Chemie
IMAR system	Distributed system (space foresight)	BNSC/Astrium, plus partners

Flown systems identified by italics.

selection of typical systems has been analysed to provide supporting data for the work presented herein, and this is shown in Table 1, below. The list includes wheeled, legged, and tracked examples, and also one distributed system.

From the list of systems shown in the table, three variants have been selected to illustrate the results obtained when the techniques discussed are applied to specific examples. The three systems chosen are:

- Nanokhod,
- Marsokhod,
- A generic rocker bogie system (similar to that used for Sojourner).

2.1. Nanokhod

Nanokhod, Fig. 1(a) and (b), is a novel design originating in Russia, although the approach has undergone substantial alteration. This was originally a novel, crawler-type of vehicle, which propelled itself in a flip–flop motion by alternate rotation and repositioning of its three body segments. This was evolved by the European Space Agency (ESA), in collaboration with Max-Planck Institut für Chemie (MPICh), and von Hoerner & Sulger GmbH (vH & S) into a tracked vehicle that moves by differential operation of the tracks, as is conventional with such systems. The instrument payload is carried in the central body which now retains its original rotary capability purely for the sake of scientific instrument positioning, rather than for the original, locomotion purposes.

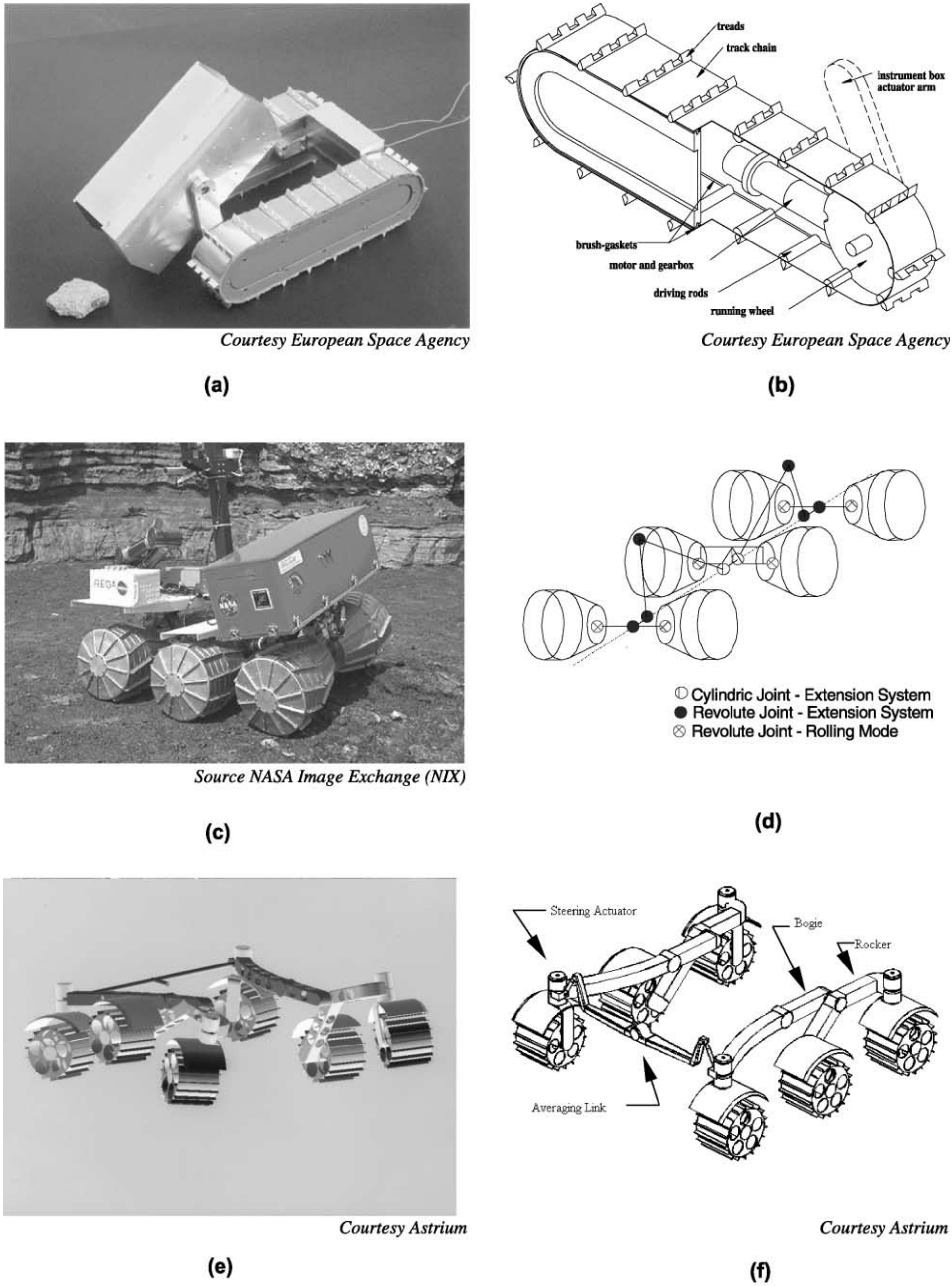

Fig. 1. (a) Nanokhod in its revised form. (b) Nanokhod sponson mechanisms. (c) The Marsokhod. (d) Marsokhod joints arrangement. (e) A rocker bogie system. (f) Rocker bogie system elements.

2.2. Marsokhod

The second illustrative system is the "Marsokhod", typical of a number of similar systems built in the USA and Europe, but originating in a concept generated by Russia's Lavochkin organisation, and developed in collaboration with NASA. The system concept, which has been adopted as the starting point for several subsequent variants, is shown in Fig. 1(c) and (d).

The Marsokhod is an extremely flexible and capable system built around an articulated, three-part chassis, with the body/payload being mounted in a distributed fashion on the various chassis elements. The front and rear wheels are mounted on chassis sections capable of lateral rotation (roll) relative to the central portion. The wheelbase is not fixed, but can be shortened or extended by means of hinge mechanisms incorporated in the front and rear chassis sections. Each of the six conical wheels is individually powered.

2.3. Rocker bogie system

The concept of the rocker bogie locomotion system has undergone extensive development by NASA/JPL, including a number of flight standard and development models. The Sojourner vehicle placed onto the surface of Mars in July 1997 as part of NASA's highly successful Mars Pathfinder mission was a product of this activity and also utilised a locomotion subsystem of the "rocker bogie" design. Because there are a number of variants of this system, a generic design is described, the principles of which are illustrated in Fig. 1(e) and (f).

This particular locomotion system is characterised by a single, rigid body chassis which is mounted to the suspension system at three positions. Two of these mounting points, one at the mid point of each side of the body, interface via revolute joints to a bogie which carries at its rear extremity a steerable, individually motorised, wheel. At the forward end, the bogie carries a further revolute joint connecting to a rocker arm which in turn carries two more wheels, each motorised, but with only the forward one of the pair being steerable. The final element in this generic design is the averaging link, which is attached to the rear of the body via a revolute joint. The free ends of the averaging link connect to the two main bogies through two small interface links.

In the following sections we use these three systems to illustrate several methodologies for describing and defining the kinematics of such devices.

3. Characterisation of kinematic systems

The classification and characterisation of kinematic structures has been the subject of considerable attention over many years. For the purposes of representing systems such as those under consideration here, a number of candidate methodologies and associated concepts based on Graph Theory and allied topics can be identified. Although reference is made to several of the underlying principles of Graph Theory, it is not considered appropriate to reproduce complete derivations and definitions of all relevant terms here. For this, the reader is directed to the references identified. The objective here is to concentrate on whether such methodologies can be useful in the classification of planetary surface locomotion systems. One of the major considerations is the extent to which such methods, either individually or in combination, can provide a unique characterisation of any one system, and whether this can adequately indicate underlying system behaviours, and their usefulness or otherwise. We make some general observations and provide working definitions of some of the terms used, before proceeding.

3.1. Kinematic chains, direct graphs and interchange graphs

The fundamental step in applying graph theory to the kinematic chain of a locomotion subsystem is the generation of its interchange graph representation, where the links and joints of the chain are represented by vertices and edges, respectively, Rooney and Earl [3]. Corresponding direct graphs are not discussed: the steps in generating interchange graphs are assumed without further comment.

It should be noted that the interchange graphs of the various systems discussed are shown for clarity without representation of the ground links. However, it should be recognised that complete analysis would require this, plus representation of ground movement due to stone rolling, soil compaction, and similar effects.

3.2. Kinematic mobility

The concept of kinematic mobility, which represents the range of movement available to any specific kinematic system is of considerable interest in the analysis of planetary locomotion systems, 'kinematic design' [4]. The mobility of a three-dimensional spatial, kinematic, system is given by:

$$M = 6(n-1) - 5j_r - 5j_p - 4j_c - 3j_s, \tag{1}$$

where n is the total number of links, and j_r, j_p, j_c, and j_s are the total numbers of revolute, prismatic, cylindric and spherical joints, respectively.

3.3. Spanning trees and spanning forests

In graph theory terminology, a forest is a disconnected graph that does not contain any cycles. A connected forest is a tree. The spanning tree of a system is the minimum subset connecting all vertices, such that no edges can be removed without the graph becoming disconnected. By the same token, a spanning forest is a forest in which removal of any edge will result in the forest becoming further disconnected, Wilson [5].

3.4. Fundamental cycles and fundamental cutsets

The fundamental set of cycles of a graph, *G*, relevant to any one of that graph's spanning trees, *T*, is the set of cycles produced by adding in turn each excluded edge of *G* to *T*. A cutset of *G* is a set of edges whose removal from *G* increases the number of components of *G*, i.e., if *G* is initially connected, then two components are produced, and if *G* is already disconnected, the number of components is increased by one. If the graph *G* is a spanning tree, *T*, which is subsequently divided into two parts, V1 and V2 by removal of an edge, then the fundamental cutset is the set of all edges joining a vertex in V1 to a vertex in V2, Wilson [5].

$$\text{Number of fundamental cycles} = m - n + 1,$$
$$\text{Number of fundamental cutsets} = n - 1,$$

where n is the number of vertices in the original graph, and m is the number of edges in the original graph.

3.5. Adjacency matrices and incidence matrices

The adjacency matrix is defined as the $n \times n$ matrix whose ijth term is the number of edges joining vertex i and vertex j, Wilson [5]. Similarly, the incidence matrix is defined as the $n \times m$ matrix whose ijth term is 1 if vertex i is incident to edge j, and 0 otherwise. Note that the numbering of the interchange graph vertices and edges does not matter, since equivalent matrices will result from any consistent approach. Equivalence can be demonstrated by showing that any adjacency matrix of a system can be derived from any other for the same system by transposition of any permutation of rows followed by the same permutation of columns, Yan and Hall [6]. In this paper, the concept of incidence matrices is not utilised, although there may be potential for future investigation in this area.

In the type of system being considered here, the terms of the various adjacency matrices are always unity, since this is typical of the types of connection being considered. Nonetheless, it ought to be borne in mind that based on the above definition, this need not be the case.

3.6. Degree sequence

The degree sequence of a graph is the sequence generated by sequential enumeration of the degrees (i.e., number of connections) of the system vertices, Wilson [5]. Conventionally one commences with the lowest degree, and works up.

3.7. Characteristic polynomials and their coefficients

The concept of characteristic polynomials and their coefficients is very powerful, and is used as a fundamental building block for the work discussed in this paper.

Yan and Hall [6] define the characteristic polynomial, P, of an adjacency matrix, A_m, as the determinant of the characteristic matrix, C, where $C = xI - A_m$, where x is a dummy variable, and I is a unit matrix of the same order as A_m. Because the coefficients of the characteristic polynomial have specific physical meaning, they can be evaluated by the "coefficients by inspection" approach suggested by Yan and Hall, as follows:

- Leading coefficient, p_0, always unity.
- Second coefficient, p_1, always zero.
- Third coefficient, p_2, is the negative of the number of connected pairs in the chain.
- Fourth coefficient, p_3, is the negative of twice the number of three pair loops.
- Fifth coefficient, p_4, is the number of two separated connected-pairs, minus twice the number of four pair loops.
- Sixth coefficient, p_5, is the negative of twice the number of five-pair loops, plus twice the number of groups formed by one three-pair loop and one connected pair.
- Higher coefficients also have physical meaning – Yan and Hall [6].

In practice, generation of characteristic polynomials by inspection is not a realistic option for systems with more than minimal complexity, and proprietary software packages such as Maple or Mathcad provide a more suitable means of extracting the required expressions.

3.8. Isomorphic and cospectral systems

Where the graph connectivity of two systems is identical, then they can be regarded as isomorphic, Uicker and Raicu [7]. If two systems have the same characteristic polynomial, then they are cospectral. Cospectral systems are not necessarily isomorphic, Harary et al. [8].

4. Derivation of representations for three typical systems

Earlier, we identified and described three typical examples of kinematic systems. By applying the characterisation approaches discussed in the previous section to these three systems, we can generate the representations shown below.

Note: In the diagrams below, notation is used as follows:

C – cylindric joint (undriven),
S – spherical joint (undriven),
R – revolute joint (undriven),
R(D) – revolute joint (driven).

4.1. Nanokhod

Nanokhod interchange graph: Fig. 2 shows the interchange graph for the Nanokhod. By inspection, the Nanokhod interchange graph is itself a spanning tree.

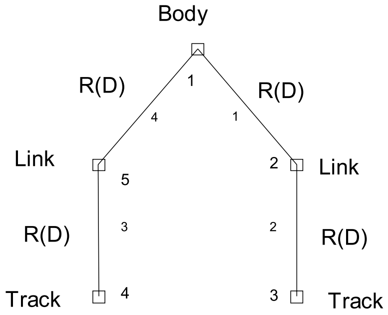

Fig. 2. Nanokhod interchange graph.

Degree sequence: The degree sequence of the Nanokhod interchange graph is $(1, 1, 2, 2, 2)$.

Kinematic mobility: Applying the standard mobility formula (Eq. (1)) to systems having only revolute joints, we can see that for the Nanokhod, the mobility, $M = 4$.

Adjacency matrix

$$A_m = \begin{vmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{vmatrix}. \tag{2}$$

Characteristic polynomial

$$P = |xI - A_m| = \begin{vmatrix} x & -1 & 0 & 0 & -1 \\ -1 & x & -1 & 0 & 0 \\ 0 & -1 & x & 0 & 0 \\ 0 & 0 & 0 & x & -1 \\ -1 & 0 & 0 & -1 & x \end{vmatrix}. \tag{3}$$

$$\text{Thus,} \quad P = x^5 - 4x^3 + 3x. \tag{4}$$

Fundamental cycles and cutsets: Since the graph is already a spanning tree, the system contains no cycles, and hence has no fundamental cycles. However, it does have four fundamental cutsets of the single-edge, bridge type.

4.2. Marsokhod

Marsokhod interchange graph: As with the Nanokhod, the graph of the Marsokhod is also itself a spanning tree (see Fig. 3). Further, no other spanning tree can be developed for the system, without first introducing additional edges that were not in the original configuration.

Degree sequence: The degree sequence of the Marsokhod interchange graph is $(1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 4)$.

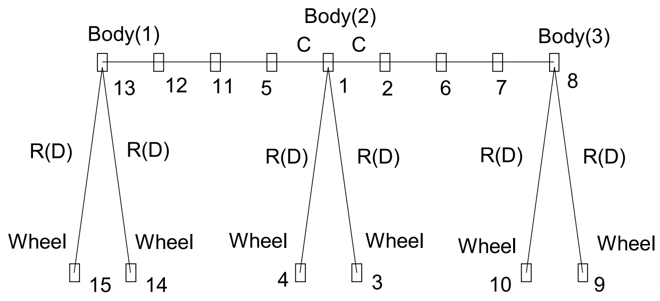

Fig. 3. Marsokhod interchange graph.

Kinematic mobility: From the standard formula, (Eq. (1)), $M = 16$.

Adjacency matrix

$$A_m = \begin{vmatrix} 0&1&1&1&1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&1\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \end{vmatrix} \tag{5}$$

Characteristic polynomial: Evaluating using Maple, we have:

$$P = x^{15} - 14x^{13} + 73x^{11} - 174x^9 + 185x^7 + 70x^5. \tag{6}$$

Fundamental cycles and cutsets: Again, the graph is itself a spanning tree, and contains no cycles, and thus no fundamental cycles. The system has 14 fundamental cutsets of the single-edge, bridge type.

4.3. Generic rocker bogie

Rocker bogie interchange graph: Unlike the two previous examples, the generic rocker bogie system being considered incorporates an averaging link, plus two "wishbones" attaching to the bogies via spherical joints. This introduces two cycles into the structure (see Fig. 4).

Degree sequence: The degree sequence for the rocker bogie system is: $(1, 1, 1, 1, 1, 1, 2, 2, 3, 3, 3, 3, 4, 4)$.

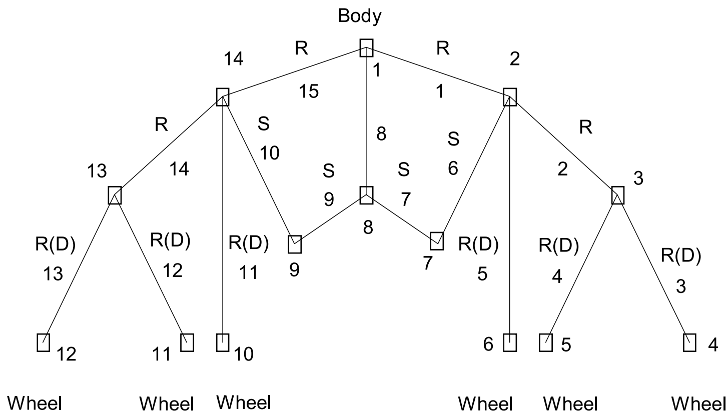

Fig. 4. Generic rocker bogie interchange graph.

Kinematic mobility: The formula, Eq. (1), is applied as for structures with only spherical and revolute joints, giving a mobility, $M = 11$.

Adjacency matrix

$$A_m = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{vmatrix} \quad (7)$$

Characteristic polynomial: Using Maple, we find:

$$P = x^{14} - 15x^{12} + 75x^{10} - 153x^8 + 124x^6 - 32x^4. \quad (8)$$

Fundamental cycles and cutsets: This system can be shown to have two fundamental cycles, and 13 fundamental cutsets.

5. Presentation of data

Given the data derived for our three typical systems in the previous section, it can be seen that a similar approach can be adopted for each of the designs previously identified in Table 1. With this information to hand, it is possible to move on to consider how best to present the results such that meaningful inferences can be derived.

Clearly, the techniques which are now discussed become increasingly useful, the larger the sample of system data which is generated. Although the data sample being considered here is comparatively small, it is considered sufficient to provide a sound basis for inferring the general usefulness of the approaches discussed, and any future developments.

A number of ways of presenting the data exist – two possibilities are discussed here – variation in polynomial coefficients, and in system degree sequence.

5.1. Plotting polynomial coefficients

The physical designs of the systems being considered, and thus their characteristic polynomials, vary widely, and appropriate data presentation methodologies need to be identified. The data can be considered to populate a 'Hilbert space' where the maximum number of dimensions is driven by the degree, n, of the characteristic polynomial derived for the most complex of the various systems analysed. Since this can be any integer, the dimension of the space of coefficients is countably infinite, hence a Hilbert space.

Although the multi-dimensional nature of the data space precludes simultaneous presentation of all the data, subsets can be plotted to represent the relationship between parameters of interest. Thus, a column graph can be constructed with the x and y axes defining which of the Hilbert space dimensions are being considered, and with the height (z axis) of each of the columns representing the scalar value for the particular variable under consideration. For example, Fig. 5 shows the variation in system mobilities with polynomial coefficients p_2 and p_4.

Thus many permutations can be envisaged for representing the multi-dimensional extent of the data space as a set of three-dimensional visualisations.

5.2. Plotting degree sequences

The second alternative approach suggested for the data to hand is to plot the degree sequences for the various systems. This can be achieved by representing the degree sequence for any particular system as a two-dimensional column graph with the columns representing the scalar degree values. The column graphs for each system can then be displayed adjacent to each other separated along a third, horizontal axis.

The result of this approach is presented in Fig. 6. The data is augmented by the inclusion of the relevant system mobilities at the left extremity of each sequence. Thus the plot provides an alternative representation of system construction and behaviour, complementary to that discussed earlier based on polynomial coefficients.

It will be noted that in this example, the order in which the systems are presented is largely arbitrary, and based on an intuitive view of the most appropriate sequence in which to order the systems. In this case the sequence is ordered by increasing

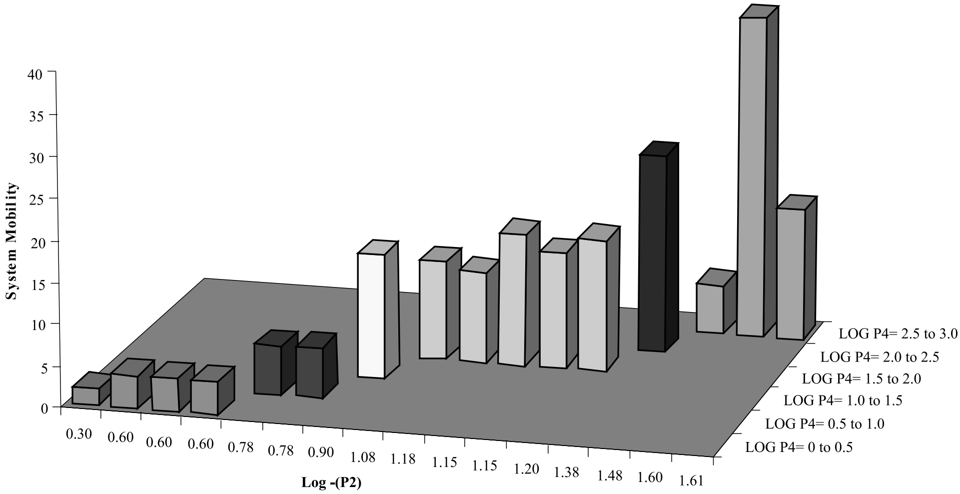

Fig. 5. The relationship between p_2, p_4 and mobility.

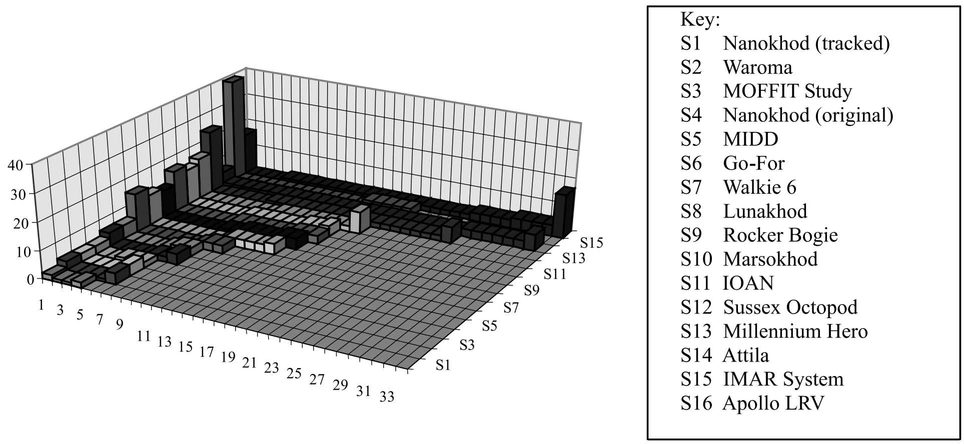

Fig. 6. Pictorial representation of system mobility and degree sequence.

number of vertices and, where equal numbers of vertices are present, 'lexicographically'.

5.3. A metric for comparing systems

As stated, a number of options exist for presenting the information available. Two possible formats have been discussed – variation in polynomial coefficients, and variation in system degree sequence. Within these formats, subsidiary options exist regarding how to treat system separation.

Table 2
Candidates for deriving inter-system 'distance'[a]

(a)	Distance, d, is calculated on the basis of a Euclidean (elliptic) metric summing the squares of the differences of the coefficients	$d^2 = (p_0 - q_0)^2 + (p_1 - q_1)^2 \cdots + (p_n - q_n)^2$	(9)
(b)	Distance, d, is derived using a 'taxicab' metric.	$d = \lvert p_0 - q_0\rvert + \lvert p_1 - q_1\rvert \cdots + \lvert p_n - q_n\rvert$	(10)
(c)	Distance, d, is calculated as for (a) using a Pseudo–Euclidean (hyperbolic) metric, subtracting the squares of the differences of the coefficients.	$d^2 = (p_0 - q_0)^2 - (p_1 - q_1)^2 \cdots - (p_n - q_n)^2$	(11)

[a] Where p_n is a coefficient within one system, and q_n is the corresponding coefficient within a second system to which it is being compared.
Expression (11) will yield a complex number for d if $[+(p_1 - q_1)^2 \cdots + (p_n - q_n)^2]$ is greater than $(p_0 - q_0)^2$.

We need to have a measure of how similar or dissimilar two systems are. We choose to use a 'distance' measure, or metric for any pair of systems, based on the polynomial coefficients. Three different metrics are considered, as shown in Table 2.

Table 3
A comparison of inter-system distance metrics

System name	Euclidean		Taxicab		Pseudo–Euclidean	
	Log_{10} distance	Sequence	Log_{10} distance	Sequence	Log_{10} distance[a]	Sequence
Nanokhod (tracked)	0.00	1	0.00	1	0.00	1
Waroma	0.30	2	0.30	2	0.30	2
MOFFIT study	0.30	3	0.30	3	0.30	3
Nanokhod (original)	0.56	4	0.70	4	0.56	4
MIDD	0.92	5	1.11	6	0.92	5
Go-For	0.95	6	1.08	5	0.95	6
Walkie 6	1.21	7	1.32	7	1.21	7
Lunakhod	2.02	8	2.27	8	2.02	8
Rocker bogie	2.33	9	2.60	9	2.33	9
Marsokhod	2.44	10	2.71	10	2.44	10
IOAN	2.57	11	2.88	11	2.57	11
Sussex octopod	2.76	12	3.11	12	2.76	12
Millennium hero	4.49	13	4.86	13	4.49	13
Attila	4.76	14	5.04	14	4.76	14
IMAR system	5.73	15	6.13	15	5.73	15
Apollo LRV	5.93	16	6.35	16	5.93	16

[a] As stated in Table 2, this expression yields a complex number where $[+(p_1 - q_1)^2 \cdots + (p_n - q_n)^2]$ is greater than $(p_0 - q_0)^2$. Yan and Hall [6] indicate that the coefficients p_0, q_0, etc. will always be unity, hence the term $(p_0 - q_0)^2$ will always be zero, and the condition will always be satisfied. Thus the distances derived from the Euclidean and Pseudo–Euclidean methods will be the same, except that the Pseudo–Euclidean distance is imaginary.

Table 4
Inter-system distances derived using the Euclidean metric

	Nanokhod (tracked)	Waroma	MOFFIT study	Nano-khod (original)	MIDD	Go-for	Walkie 6	Luna-khod	Rocker bogie	Marso-khod	IOAN	Sussex octopod	Millen-nium hero	Attila	IMAR system	Apollo LRV
Nanokhod (tracked)		0.30	0.30	0.56	0.92	0.95	1.21	2.02	2.33	2.44	2.57	2.76	4.49	4.76	5.73	5.93
Waroma			0.00	0.48	0.88	0.92	1.19	2.02	2.33	2.44	2.57	2.76	4.49	4.76	5.73	5.93
MOFFIT study				0.48	0.88	0.92	1.19	2.02	2.33	2.44	2.57	2.76	4.49	4.76	5.73	5.93
Nanokhod (original)					0.69	0.73	1.10	2.02	2.33	2.44	2.57	2.76	4.49	4.76	5.73	5.93
MIDD						0.35	0.93	2.00	2.32	2.43	2.57	2.76	4.49	4.76	5.73	5.93
Go-for							0.86	2.01	2.32	2.43	2.57	2.76	4.49	4.76	5.73	5.93
Walkie 6								2.00	2.32	2.43	2.57	2.76	4.49	4.76	5.73	5.93
Lunakhod									2.05	2.26	2.47	2.71	4.49	4.76	5.73	5.93
Rocker bogie										1.87	2.31	2.64	4.49	4.76	5.73	5.93
Marsokhod											2.15	2.58	4.49	4.76	5.73	5.93
IOAN												2.38	4.49	4.76	5.73	5.93
Sussex octopod													4.48	4.75	5.73	5.93
Millennium hero														4.53	5.74	5.92
Attila															5.74	5.92
IMAR system																5.99
Apollo LRV																

Any of these relationships can be used to derive inter-system distance. In the simplest case, this can be calculated but only used simplistically to define the sequence in which the systems are presented within the plot. This approach has the advantage that readily available automatic chart generating systems such as those available in spreadsheets can be employed, without recourse to more powerful tools.

Alternatively, any set of distances derived using one of the approaches stated in Table 2, can be used to define actual positioning along the third axis relative to a 'base' system. This approach has the advantage that any future development of this technique relying on the actual surface contours of any carpet plots produced would be based on truly representative surfaces, not on surfaces constructed using artificial separations. Furthermore, adopting inter-system distance as the third axis would indicate relative system complexity, and may be a useful indicator of relative system capabilities.

At this stage, it is appropriate to consider the inter-system distances obtained for the 16 baseline systems identified in Table 1. The distances (expressed logarithmically) resulting from Eqs. (9)–(11) (Table 2), are shown in Table 3. Also shown are the system sequences obtained from each of these approaches.

As can be seen from the table, although minor differences exist, the results are substantially the same for all three metrics. Having established this point, we can proceed to adopt one metric, and to evaluate the full set of inter-system distances for our sample of 16 systems. These are presented in matrix form in Table 4, using the Euclidean metric. Only half the matrix is stated, since it is symmetric about the leading diagonal. The leading diagonal itself is omitted since this is all zeroes, representing the distance between each system and itself.

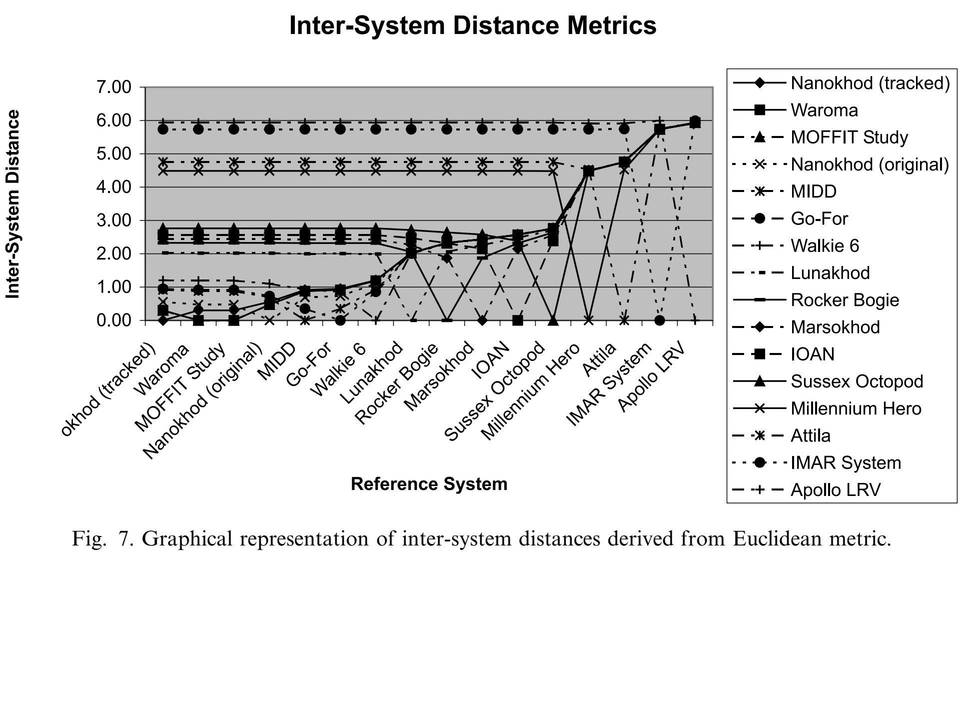

Fig. 7. Graphical representation of inter-system distances derived from Euclidean metric.

The data from Table 4 is presented graphically in Fig. 7. This presentation allows cross-referencing between any two systems to be carried out, and the respective inter-system distance to be derived.

Thus it can be seen that the potential exists for systems to be characterised, albeit incompletely, in a way which facilitates their graphical representation and comparison. It is also possible to apply exactly the same techniques to the representation of different states of fault or failure existing within a single system, instead of different systems. Indeed, where fault or failure conditions of one system yield characteristics very similar to another system, hybrid representations can be envisaged allowing fault and failure conditions of several systems to be compared.

However, before exploring any of these possibilities further, it is necessary to consider why graph theory provides such a useful tool for the representation of kinematic system behaviour. The following section discusses the way in which fault and failure conditions can be treated using this method.

6. Trees, fundamental cycles and fundamental cutsets as a route to robustness

In order for a design methodology for complex mechanical architectures to be constructed based on graph theory approaches, the question of how fault and failure modes are identified and dealt with needs to be tackled. It can be seen that the concept of fundamental cycles and fundamental cutsets can provide an insight into the robustness to faults and failure of a system. If it is taken as read that the penultimate stage (fault condition) in system degradation prior to failure may be represented as a spanning tree, then clearly robustness can be built into a system by building in cycles. In order to arrive at the desirable state of system design which provides graceful degradation in the face of continuing faults, an approach needs to be identified which provides for successive (possibly dormant?) cycles to become operative as the system degrades. If a link can be created between the theoretical methodology for achieving this, and practical system realisation, then the basis of a valid design tool can be established.

Thus, we can postulate that a particular mechanical architecture could be represented by a *complete* (interchange) graph, G, such that the total complement of edges (joints), $E(G)$ and vertices, (links), $V(G)$, together encompasses the entire, potentially available, functionality/range of operational modes of the device.

Given the representation of the target architecture by $G = \{V(G), E(G)\}$, it can be further postulated that there must exist a series of non-disconnecting sets, $\{e_m \cdots e_n\}(G)$, which represents the total number of possible fault modes of the system which can occur without total system failure. When any permutation of $\{e_m \cdots e_n\}(G)$ acts on G, the resulting incomplete graph, $\Delta(G)$, will represent the connectivity/functionality of the system when operating in the resulting fault condition.

The situation will, however, be complicated by the dynamic nature of the underlying problem. The locomotion of any autonomous system involves the continual opening and closing of kinematic loops as the wheels/tracks/legs of the system make and break contact with the surface. In this situation, the incomplete graph, $\Delta(G)$, resulting from the intervention of a non-disconnecting set, $\{e_m \cdots e_n\}(G)$, must continue to represent a viable system configuration as the kinematic loops open and close due to system locomotion. Provided that this is the case, then this set represents a fault mode from which recovery is possible – if not, then it would represent a failure condition.

Given the definition of a tree as a minimally connected graph where the removal of any edge will make it disconnnected, then clearly if a set of mechanisms is operating as a tree, any one fault will cause overall system failure. Consequently, for system robustness to be achieved, then G must be capable of being acted upon by several elements of $\{e_m \cdots e_n\}(G)$ before disconnection occurs. This clearly corresponds directly to classical redundancy thinking.

In the situation where G is acted upon by a cutset, $C(G)$, then this situation will result in failure of the system. Thus it can be seen that there will be a set of spanning trees, $T(G)$, for the system, which represent various conditions of operation with nil redundancy. There will be a corresponding set of cutsets, $C(G)$, the action of which will result in catastrophic failure (occurring when the graph has become disconnected).

Clearly, any linkage whose interchange graph is a spanning tree is not robust to kinematic/mechanical faults. The challenge must be to connect the graph to the maximum extent possible in order to maximise fault tolerance. The level of reconnection will be directly influenced by practical considerations, and real architectures will exhibit a finite number of alternative operational modes.

This may be seen as a constraint on the generality of any solution, and on the flexibility which can be brought to the system by advanced software techniques, and hence as a fundamental flaw. Genetically derived code, for example, can encompass very large numbers of alternative strategies to achieve a particular goal, and it may be considered that this capability could not be matched by any graph representation with a realistic level of complexity. However, the factorial nature of permutations means that even with finite numbers of operational modes, considerable flexibility will still exist within the system. This will enable it to capitalise on the inherently indeterminate nature of (e.g.) the evolved control software solution, and, in practice, this is not considered a fundamental weakness.

7. Outcome

We have demonstrated that it is feasible to establish schemes whereby locomotion systems can be analysed and classified, and the results presented graphically. However, the scheme adopted does not provide for entirely unique identification of individual systems – the problem of kinematic isomorphism (see Kong et al. [9] for useful review of work in this field), remains to be satisfactorily catered for.

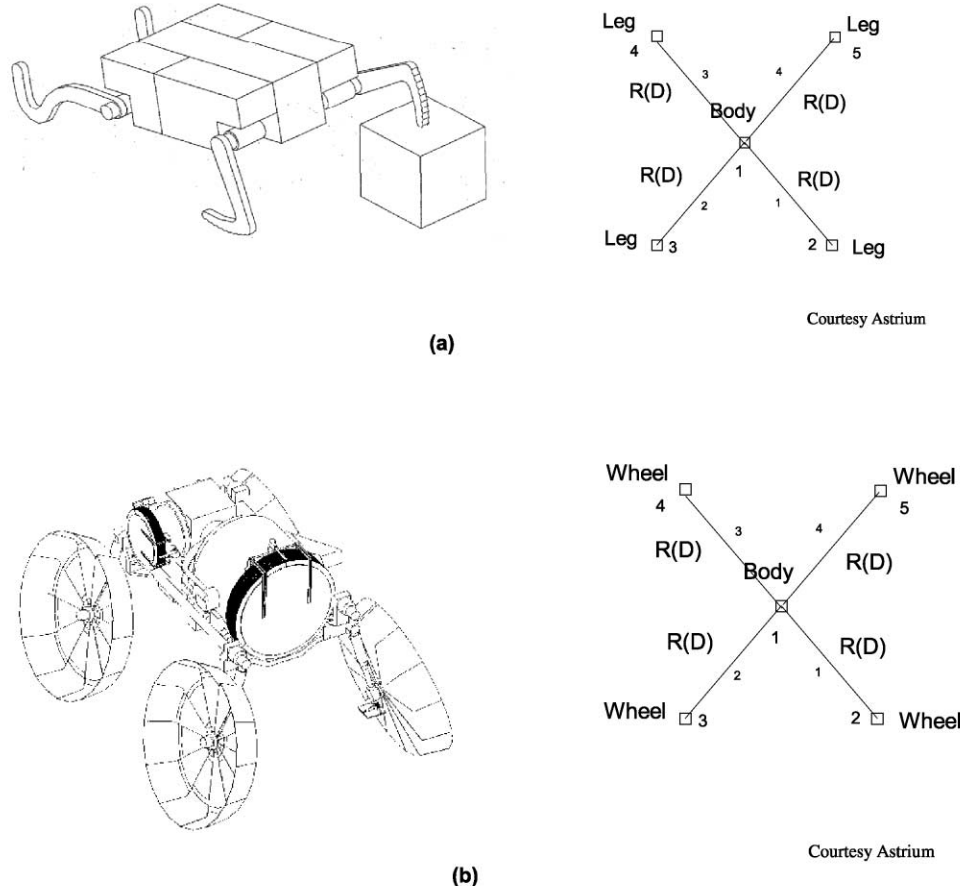

Fig. 8. (a) A ‘flipper’ locomotion system. (b) MOFFIT lunar vehicle.

At this stage, it is worth revisiting the methodology by which the systems have been analysed. So far, the analysis has been carried out on a strictly kinematic basis, ignoring geometric and engineering differences. Where it is practicable for such differences to be included in the proposed classification, this would allow the generation of more comprehensive descriptions of systems, and thus differentiation between systems which were otherwise indistinguishable on a purely kinematic basis (i.e., isomorphic).

Consider, for example, the graphs of the two design studies illustrated in Fig. 8 – a ‘flipper’ type of locomotion system, not unlike ESA’s Waroma concept, and a wheeled vehicle designed as part of the Moon based free flyer interferometer trade-off (MOFFIT) study. In both instances the graph is a simple four-branched tree comprising five links, four joints, and having a kinematic mobility of 4. (The MOFFIT vehicle has other degrees of freedom, but these are for deployment purposes only, and are not strictly part of the locomotion system.) Clearly, this is not the whole story – the methods of locomotion employed are fundamentally different – in one case contact is retained with the ground by the wheels, and in the other, the legs (flippers) make only intermittent contact.

Here we see a fundamental discriminator. In the first case there is nominally unbroken ground contact where the drive system employs continuous rotation of its revolute joints in one direction only (for a given direction of vehicle motion). In the other case, it is clear that the appendages of legged or flipper systems may make only intermittent contact with the ground. In this latter case, the revolute joints may or may not be required to make complete rotations. In the case of a true leg, the mechanism is able to employ either prograde or retrograde movement whilst the vehicle maintains one direction of travel. (There is, however, a difference between the flipper action of the system illustrated, and the action of a true leg. In the example chosen, the flippers employ continuous rotation, and might more accurately be regarded as rudimentary wheels.)

Thus we see that a modification of the approach described earlier could be adopted. In several of the diagrams used for illustrative purposes in this paper, graph labels have been employed, although no use has been made of this facility as a consistent element of the methodology. We now see that labelling offers a convenient way of identifying the geometric and engineering differences discussed above, and would provide a useful additional discriminator.

If labelling was to be included as an element of the classification scheme, this could, for example, be used to improve the expression defining separation distance along the third axis of the graphs discussed earlier. It should be possible to derive an expanded distance expression providing a better representation of actual mechanism family groupings. If the labels used were also suitably weighted, they would then provide a powerful means of evolving graphical representations in which the graph topology was more indicative of the locomotion method employed.

Thus we can see that any of the expressions for distance considered earlier could be modified to take account of the actual physical characteristics of the system. One of many possibilities might be the inclusion of additional, empirical, factors, g_i, dictated by the system geometry/method of operation. This could result in a modified Euclidean metric, as follows:

$$d^2 = \sum_{i=0}^{n} g_i (p_i - q_i)^2.$$

Further investigation of these possibilities remains as a potentially fruitful area of investigation in which a number of variations on the theme of enriching the distance expression might be evaluated. However, it is worth noting that since taking account of dynamic system characteristics involves considerations of ground contact, gait, etc., then it may prove necessary to introduce graph theory representations of ground contact into the analysis before adequately descriptive system characterisations can be achieved.

8. Concluding remarks

Investigations to date have been sufficiently detailed for there to be a reasonable level of confidence that graph theory methodologies and associated theory have much to offer in the classification of planetary exploration systems. The methods of representation investigated hold out the promise that workable design theories can be extracted from this approach.

The desirable outcome of the work is to arrive at a means of consolidating system design parameters into a single, integrated data structure, or at least into a coherent set of related structures. If this can be achieved, then the target characteristics for a new system can be used to position the new design within the data structure, thus allowing the requisite parameters of the new system to be identified.

Future work will include detailed investigation into methodologies for synthesis of new system designs based on the analysis of actual designs in operation, and utilising the groundwork described within this paper. This will include investigations into the synthesis of complex system graph characteristics from simpler, subsystem, representations. Also to be covered are methods for producing variants to a baseline system morphology such that fixed mobility criteria can continue to be met as system connectivity is modified. This should allow fresh insight into ways of catering for system faults and failures.

Acknowledgements

The authors would like to thank ESTEC for permission to reproduce the illustrations of the Nanokhod, and Astrium for permission to publish this paper. The opinions expressed within this paper are those of the authors, and not necessarily those of Astrium.

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